MODELING A BENT TWISTED FILAMENT IMMERSED IN FLUID

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This paper is concerned with modeling the interaction between an immersed elastic filament and the surrounding fluid, within the framework of the immersed boundary method [1]. As a result of the fluid motion, the filament undergoes large deformations in three spatial dimensions, and it applies forces and moments to the fluid that resist the stretching, shearing, bending, and twisting of the filament. The filament motion is described by the centerline $\mathbf{X} = \mathbf{X}(s,t) : I\!R \times I\!R \to I\!R^3$ parameterized by arc length s and time t, and a local triad $(\mathbf{D}_1(s,t), \mathbf{D}_2(s,t), \mathbf{D}_3(s,t))$ associated with this curve [2]. With the internal forces and moments expressed in $(\mathbf{D}_i)_{i=1,2,3}$ basis, $\mathbf{F}(s,t) = \sum_{i=1}^3 F_i \mathbf{D}_i$, $\mathbf{M}(s,t) = \sum_{i=1}^3 M_i \mathbf{D}_i$, and $\mathbf{f}$, $\mathbf{m}$ the interaction forces and moments with the fluid, the equilibrium equations read

$$\mathbf{0} = \mathbf{f} + \frac{\partial \mathbf{F}}{\partial s}, \quad \mathbf{0} = \mathbf{m} + \frac{\partial \mathbf{M}}{\partial s} + \frac{\partial \mathbf{X}}{\partial s} \times \mathbf{F}. \tag{1}$$

To complete these equations, we introduce constitutive relationships, which take the form

$$F_i = b_i\left(\frac{\partial \mathbf{X}}{\partial s} - \mathbf{D}_3\right) \cdot \mathbf{D}_i, \quad M_i = a_i \frac{\partial \mathbf{D}_j}{\partial s} \cdot \mathbf{D}_k, \tag{2}$$

where (i,j,k) is any cyclic permutation of $(1,2,3)$, and a_i and b_i with $i = 1,2,3$ are material constants. For a given configuration of the fiber $\{\mathbf{X}, (\mathbf{D}_i)_{i=1,2,3}\}$, Eqs. (1) provide enough information to compute $\mathbf{f}$ and $\mathbf{m}$ to be spread into the fluid domain. The process of "spreading" involves the $3D$-Dirac delta function [1] and its gradient. The influence of the filament on the fluid is expressed in terms of a singular body force $\mathbf{f^b}$, which is given by

$$\mathbf{f^b}(\mathbf{x},t) = \int \mathbf{f}(s,t)\delta(\mathbf{x} - \mathbf{X}(s,t))\, ds + \int -\frac{1}{2}\mathbf{m}(s,t) \times \nabla\delta(\mathbf{x} - \mathbf{X}(s,t)) ds. \tag{3}$$

A procedure dual to the above updates the position of the curve and the triad by interpolating the fluid velocity $\mathbf{u}$ and the angular velocity $(\nabla \times \mathbf{u})/2$:

$$\frac{d\mathbf{X}}{dt} = \int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x} - \mathbf{X}(s,t))\, d\mathbf{x}, \quad \frac{d\mathbf{D}_i}{dt} = \frac{1}{2}\int (\nabla \times \mathbf{u})(\mathbf{x},t)\delta(\mathbf{x} - \mathbf{X}(s,t))\, d\mathbf{x} \times \mathbf{D}_i. \tag{4}$$

The discretization of Eqs.(1)-(4) is straightforward, using finite differences schemes with central difference operators. Numerical simulations involving open and closed filaments will be shown.

References

[1] C. Peskin, "The Immersed Boundary Method" *Acta Numerica*, p. 1-39, 2002.
[2] A. Goriely, M. Tabor, "Nonlinear dynamics of filaments II: Nonlinear Analysis" *Physica D*, v. 105, p. 45-61, 1997.